# Generic Photovoltaic System Emulator Based on Lambert ω Function

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Abstract- This paper describes the development of a generic model suitable for emulating Photovoltaic (PV) system characteristics in order to study its impacts on the performance power networks. This model can accurately emulate the dynamic behaviour of the PV system, including the Maximum Power Point Tracking (MPPT) controller. A new method to emulate the MPPT controller based on the Lambert & function is introduced. This method can accurately determine the PV array optimum operating voltage point without the need for measuring the output power. This makes the techniques suitable for practical implementation, as the response is fast and does not involves the oscillations usually associated with the Perturbation and Observation (P&O) MPPT. It also provides improved performance in low solar irradiance conditions, as compared with conventional MPPT methods. The proposed emulator is easy to implement in computer modelling of power systems. This model is implemented in Matlab/Simulink Software package and its performance is analysed and presented in this paper.

Index Terms- generic Photovoltaic system model, MPPT, Lambert ω function, dynamic behaviour emulation

### I. Nomenclature

	i. ItomEncerrote		
λ	Solar irradiance (KW/m²)		
$T_{\mathbf{C}}$	Cell operating temperature (°C)		
$I_{PH}$	Photocurrent (A)		
$I_D$	Diode current (A)		
$I_{SC}$	Short circuit current (A)		
$I_S$	Saturation current (A)		
I <sub>RS</sub>	Reverse saturation current (A)		
$\mathbf{E}_{\mathbf{G}}$	Bang-gap energy of the semiconductor (eV)		
K	Boltzmann constant		
k'	proportion constant for short-current MPPT control		
$K_{I}$	Cell's short-circuit current temperature coefficient		
q	Electron charge (C)		
Α	Ideal factor		
$N_s$	Number of cell in series		
$N_P$	Number of cell in parallel		
P	PV array output power		
V	PV array operating voltage (V)		
I	PV array output current (A)		
MPP	Maximum power point		
$P_{MPP}$	PV array maximum power output (W)		
$V_{ extsf{MPP}}$	PV array operating voltage on MPP (V)		
$I_{MPP}$	PV array output current on MPP (A)		

#### II. INTRODUCTION

Due to the concern about the levels of greenhouse gas emission and over exploitation of fossil fuel, solar energy is emerging as a clear, free, alternative power source and attracting a growing amount of political and commercial interest. For instance, the UK government forecasted that solar photovoltaic system installation capacity will grow to 30 GW by 2050 [1]. If such high penetration of PV generation is connected to the electricity network, their effects on the security of the power network must be carefully considered. For example, if the supply and demand are not balanced, the power network voltage and frequency may deviate beyond the statutory tolerance.

PV generation systems have two specific characteristcs:1) the efficiency of the PV generation is low due to the material used and manufacturing limitations and 2) the generation only occurs during the day time. Therefore the maximum power tracking is indispensable in PV generation. Thus, many MPPT control methods have been proposed. The most widely used one is the "perturbation and observation" (P&O) method, which is well known as the "hill-climbing method". This method searches for the MPP by checking the differential coefficient of the power (P) with respect to the voltage (V) or current (I) (dP/dV or dP/dI). The main drawback of this method is the oscillation that happens around the optimum voltage point. This results in loss of energy capture that increases with the step size of the perturbation. If the step size width is big, it would cause more loss. But with reducing the step size width, the response time to track the MPP would be longer [2]. Another MPPT control method called "incremental conductance method" overcomes the oscillation appearing in the ordinary P&O method. This method is based on comparing the values of conductance increment (dI/dV) with the conductance (I/V). Although this technique removes the effect of the oscillation, its performance is not good enough in lower irradiation regions, as the P-V characteristic only covers a small range of voltage [3]. Another MPPT control methodology is named "short-current", in which the optimum output current is calculated by multiplying the short circuit current by the proportional constant ( $I_{OP} = k' \times$ I<sub>SC</sub>). This technique solves the oscillation problem, but the step size remains a problem. As k' is not always constant, it would decrease in the low irradiance which can bring PV generation efficiency down [4].

Obviously, there will be different PV systems from different manufactures connected to the distribution network. Hence, for analysis of distribution networks, a generic PV system model would be useful to help simulate different PV systems behaviour and their impacts on the power network. proposed PV system emulator emulates any PV panel's behaviour based on the product's specifications. The principles of the proposed emulator may be used to develop a MPPT controller. This control method is based on the 'Lambert  $\,\omega$ function'. It overcomes the conventional MPPT methods 'drawbacks', removing the oscillation phenomena, reducing the response time and improving the PV systems' performance at low irradiance by determining the optimum operating voltage

As this PV system emulator is able to calculate the optimum voltage and maximum power directly, so it is easier to be implemented, as compared with the conventional gridconnected PV system models used in power system steady-state studies.

# III. GENERIC PHOTOVOLTAIC MODEL

# A. Solar cell model

A PV array is a group of PV cells which are connected in series-parallel configuration. Each cell is basically a p-n junction fabricated in a thin wafer or layer of semiconductor. The electricity can be converted from electromagnetic radiation of solar energy directly by the photovoltaic effect. The equivalent circuit of a solar cell consists of a photo current source, a diode, a shunt resistor (representing the leakage current) and a series resistor (representing the cell's internal resistance to the current flow). The resistors can be ignored, as their effect on the power generation is not significant [2] and this results in the ideal cell equivalent circuit shown in Fig. 1. The solar cell current output is:

$$I = I_{PH} - I_D \tag{1}$$

Where the photocurrent  $I_{PH}$ , mainly depends on the solar irradiance ( $\lambda$ ) and operating temperature (T<sub>C</sub>), can be expressed as:

$$I_{PH} = [I_{SC} + K_I (T_c - T_{ref})] \lambda$$
 (2)

The diode current ID is affected by the cell's saturation current and cell operating temperature [5]. Based on the Shockley diode equation, it can be summarized as:

$$I_D = I_S \times (e^{\frac{qV}{KAT_c}} - 1) \tag{3}$$

Is is the cell's saturation current which changes with the operating temperature. Its behaviour is described as:

$$I_S = I_{RS} \times (\frac{T_c}{T_{ref}})^{\frac{3}{A}} \times e^{\frac{qE_G}{KA}(\frac{1}{T_{ref}} - \frac{1}{T_c})}$$
 (4)

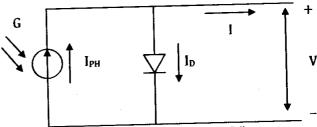


Fig. 1. Equivalent Circuit of Ideal Solar Cell

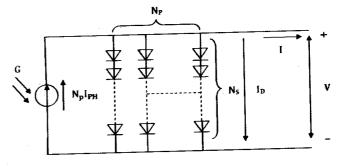


Fig. 2. Equivalent Circuit of an Ideal PV Array

IRS is the reverse saturation current. Given the open circuit voltage (Voc) at the reference temperature and ignoring the shunt-leakage current, it can be approximately obtained as:

$$I_{RS} = \frac{I_{SC}}{\frac{qV_{OC}}{qV_{OC}}} \tag{5}$$

The I-V characteristic equation of an ideal solar cell is given as:

$$I = I_{PH} - I_S \times (e^{\frac{qV}{KAT_C}} - 1) \tag{6}$$

# A. PV array model

As a typical solar cell's generation is normally less than 2 W, therefore a number of cells would be connected in seriesparallel configuration as a module. Then the modules are connected and assembled into an array which could produce sufficient power. The simplified equivalent circuit for a Np parallel and  $N_S$  series arrangement PV array is shown as Fig. 2. The I-V characteristic mathematical equation of a generalized PV array can be described as:

$$I = N_P I_{PH} - N_P I_S \left[ exp\left(\frac{qV}{N_S K T_c A}\right) - 1 \right]$$
 (7)

# IV. THE PROPOSED PHOTOVOLTAIC SYSTEM MPPT CONTROLER

The relationship between the PV array operating voltage (V) and current (I) is highly nonlinear. The combination of V and I which maximizes the efficiency of the photovoltaic energy conversion at different solar irradiance and cell temperature is the MPP. A MPPT system is consequently needed to maximize the PV power output. Here, a new MPPT control technique based on the 'Lambert ω function' is used to determine the PV

optimum operating voltage directly. This signal can then be used as the operating voltage reference.

Then assume:

$$C_1 = N_P I_{PH} = N_P [I_{SC} + K_I (T_C - T_{ref})]$$
 (8)

$$C_2 = I_{RS} = \frac{I_{SC}}{\frac{qV_{QC}}{e^{N_SKAT_{C-1}}}} \tag{9}$$

$$C_3 = N_P I_S = N_P \times I_{RS} \times \left(\frac{T_C}{T_{ref}}\right)^{\frac{3}{A}} \times e^{\frac{qE_G}{KA}\left(\frac{1}{T_{ref}} - \frac{1}{T_C}\right)} \quad (10)$$

$$C_4 = \frac{q}{N_s KAT_c} \tag{11}$$

Substituting equations (8), (9), (10) and (11) into (7), gives:

$$I = C_1 \lambda - C_3 \times (e^{C_4 V} - 1) \tag{12}$$

The PV array power output is:

$$P = V \times I = (C_1 \lambda + C_3)V - C_3 V e^{C_4 V}$$
 (13)

According to the P - V characteristic, the maximum power generation occurs at dP/dV=0. Thus, the MPP can be determined when:

$$\frac{\Delta P}{\Delta V} = C_1 \lambda + C_3 - C_3 e^{C_4 V} (1 + C_4 V) = 0 \tag{14}$$

$$1 + \frac{c_1}{c_3}\lambda = e^{C_4V}(1 + C_4V) \tag{15}$$

Multiplying both sides of equation (15) by the constant 'e', the expression can be written as:

$$e(1 + \frac{c_1}{c_2}\lambda) = e^{C_4V + 1}(C_4V + 1)$$
 (16)

Then, using the 'Lambert  $\omega$  function'[6], the optimum operating voltage can be determined as:

$$V_{MPP} = \frac{\omega\left(e\left(1 + \frac{C_1}{C_3}\lambda\right)\right) - 1}{C_4} \tag{17}$$

Substituting equations (8), (9), (10) and (11) into (17), the final expression for the optimum operating voltage is:

$$V_{MPP} = \frac{\omega \left( e^{\left(1 + \frac{|I_{SC} + K(T_{C} - T_{ref})|}{\frac{I_{SC}}{qV_{OC}} \times (\frac{T_{C}}{T_{ref}})\frac{3}{N_{A}} \times \frac{qE_{C}}{KA}(\frac{1}{T_{ref}} \frac{1}{T_{C}})} \lambda \right) - 1}{\frac{q}{N_{SKAT_{C}}}}$$
(18)

# V. PV SYSTEMS MODEL IMPLEMENTAION AND SIMULATION

On the basis of the equations derived above, the PV system model is implemented in the Matlab/Simulink software package. Here, the Solarex MSX 60 PV panel is adopted for simulation. Its specifications are shown in Table I. All these parameters are measured at standard condition (1000 W/m², 25°C).

### A. Generalized PV model

The generalized model of a PV panel, as developed in Matlab/Simulnk is shown in Fig. 3. This model can emulate different PV panels' behaviour by inputting the specifications into the dialog box and generating the I-V characteristic curve at any particular solar irradiance and cell temperature. Figs. 4 and 5 are the I-V curves which were simulated based on the scenarios given in table II.

TABLE I [7] SOLAREX MSX 60 SPECIFICATIONS

Characteristics	SPEC.
Rate power	60W
Voltage at P <sub>MPP</sub> (V <sub>MPP</sub> )	17.1V
Current at P <sub>MPP</sub> (I <sub>MPP</sub> )	3.5V
Short-circuit current (I <sub>SC</sub> )	3.8A
Open-circuit voltage (Voc)	21.1V
Temperature coefficient of	-73mV/°C
open-circuit voltage	
Temperature coefficient of	3mA/°C
short-circuit current	
Temperature coefficient of power	-0.38w/°C
Normal operating cell temperature (NOCT)	47±2°C

TABLE II

Scenario 1	
Solar Irradiance (fixed)	Cell Temperature (varied)
1 KW/m <sup>2</sup>	0° C
*	25° C
	50° C
	75° C
Scenario 2	
Cell Temperature (fixed)	Solar Irradiance (varied)
25° C	0.25 kW/m <sup>2</sup>
	0.5 kW/m <sup>2</sup>
	0.75 kW/m <sup>2</sup>
	1 kW/m²

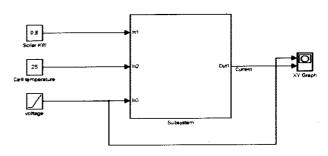


Fig. 3. Generalized PV Model

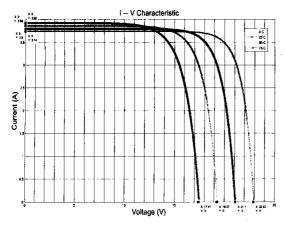


Fig. 4. I-V Output Characteristics with Different Cell Temperatures

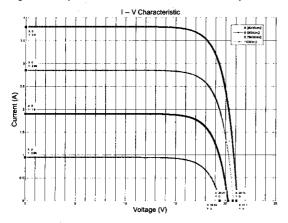


Fig. 5. I-V Output Characteristics with Different Solar Irradiances

From Figs. 4 and 5, it can be seen that the open circuit voltage decreases and short circuit current increases with the cell temperature increasing; both of open circuit voltage and short circuit current would go up if the solar irradiance increases.

### B. MPPT controller model

The MPPT controller based on 'Lambert  $\omega$  function' and described by equations (8-18) was implemented in Matlab/Simulink. The controller model determines the voltage at the maximum power point based on the nodule I-V characteristics at the given irradiance and temperature which are entered by the user, as shown in Fig. 6. Then this optimum voltage is used as a reference signal for the PV panel model, as shown in Fig. 7, which would operate at maximum power.

Fig. 8 simulated a rapid solar irradiance variation (from 0.4 kW/m2 to 0.8 kW/m2). Simulation results shown in Fig. 9 shows that there are no oscillations or delay in the system response. This is because the proposed controller determines the optimum voltage directly without the need to increase or decrease the step, as for the P&O MPPT controller. Therefore, the overall PV system's performance would be improved.

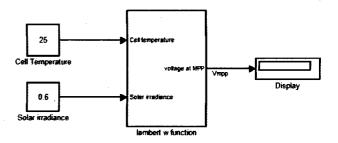


Fig. 6. 'Lambert ω function' MPPT Controller Model

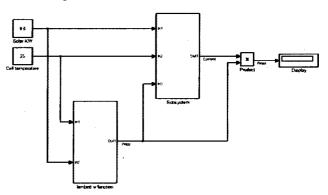


Fig. 7. PV Maximum Power Generation Model

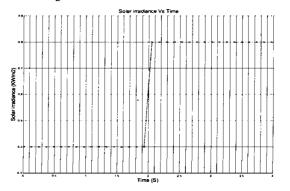


Fig. 8. A Sudden Change in Solar Irradiance

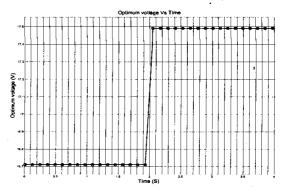


Fig. 9. Optimum Voltage Response

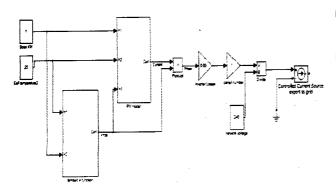


Fig. 10. Generic PV System Emulator Model

C. Generic PV system emulator model

A generic PV system emulator suitable for power network simulation has been built and tested. The PV system model, including the panel and the MPPT controller, generates the maximum power at any given irradiance and cell temperature. For power system simulation studies, the power output from a PV generation station can be emulated as output power of a single PV panel multiplied by the number of panels and the inverter efficiency. Accordingly, the current fed into the grid may be calculated by dividing the total power generated by the network voltage. The Matlb/Simulink PV system model is shown in Fig. 10.

### VI. CONCLUSION

This paper presents the details of a generic model suitable for emulating PV system characteristics in order to study its impacts on the performance power networks. The proposed model can emulate any PV panel's characteristics according to their specifications and may be implemented easily as compared to other models of PV systems. The proposed MPPT model is very simple to implement and doesn't require modelling of controllable power electronics converters, as with other types of grid-connected MPPT models. This proposed PV system emulator is suitable for steady-state power system analysis. Work is ongoing to adapt the emulator to make it suitable for simulating dynamic conditions. In this emulator, a novel MPPT control method based on 'Lambert & function' is introduced. This MPPT method overcomes the drawbacks of conventional MPPT control methods, by removing the oscillation and reducing the response time. This improves the PV generation performance at low solar irradiance.

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